

Asymptotic normality through factorial cumulants and partitions identities

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SUMMARY

We develop a relatively general approach to asymptotic normality through factorial cumulants. Factorial cumulants arise in the same manner from factorial moments, as (ordinary) cumulants from (ordinary) moments. Another tool we exploit is a new identity for "moments" of partitions of numbers. As a result we show that the asymptotic normality holds if factorial moments can be written in a suitable exponential-polynomial form. It appears that such a form is quite natural in many probabilistic models. Thus this general limiting result is used to (re)derive asymptotic normality for several models including classical discrete distributions, occupancy problems in several generalized (inverse) allocation schemes, including e.g. occupancy for distinguishable or indistinguishable balls or random forests.

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