

# Random fuzzy sets: a probabilistic tool to develop statistics with imprecise data<sup>†</sup>

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## SUMMARY

Random fuzzy sets (for short RFSs) are random elements taking on values on classes of fuzzy sets of Euclidean spaces. They have been introduced as a mathematical model for random mechanisms generating imprecise values which can be properly formalized by means of fuzzy sets.

The model involves a certain Borel-measurability assumption guaranteeing one can rigorously refer to: the induced distribution of an RFS, the stochastic independence of RFSs, a simple random sample from an RFS, and so on.

In spite of the fact that RFSs can be viewed as a non-trivial special type of functional-valued random elements, some specific features (mostly related to the arithmetic between fuzzy values) state crucial differences between them. Thus, on the basis of an isometrical embedding, fuzzy values can be identified with functional ones, although through such an identification the classes of fuzzy values are associated with closed convex cones of Hilbert spaces but not Hilbertian themselves.

In analyzing fuzzy data for statistical purposes several ideas and approaches from the analysis of real/vectorial data can be extended, but often special care should be taken in avoiding the inconveniences due to the lack of: linearity and complete ordering in handling fuzzy data; friendly and realistic models for the distributions of RFSs; Central Limit Theorems for RFSs where the limit distribution corresponds to a random element taking on values inside the associated cone. These inconveniences are ‘bypassed’ by considering a distance-based methodology using a generalized bootstrapped Central Limit Theorem.

**Keywords:** fuzzy arithmetic, metrics between fuzzy values, random fuzzy sets.

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