

The number of records in geometric samples[†]

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SUMMARY

Records are left to right local maxima of a sequence of numbers. Counting the number of records in random sequences is a problem of interest in several applied areas like Hydrology, Climatology, Finances, Computing Sciences, etc. It is well known that the distribution of the number of records in a random permutation and in the case of iid samples from an absolutely continuous parent can be written in terms of the Stirling numbers of the first kind. In the latter case it is remarkable that the distribution of the number of records does not depend on the parent distribution. For discrete parents, the situation is more complicated due to ties and to the fact that the distribution of the number of records depends on the parent distribution.

In this work, we study the distribution of the number of records (ordinary and weak) in iid samples from geometric distributions. Although some asymptotic results about these distributions have been studied before by some authors, little is known about the exact distributions. Our main result shows that the distribution of the number of records can be written in terms of the q -Stirling numbers of the first kind. We also study the exact distribution of record times and its connection to non-central q -Stirling numbers. The key for the obtention of these results is a new representation of (non-central) q -Stirling numbers of the first kind as a multiple Jackson integral.

Keywords: records, q -calculus, q -Stirling numbers, Jackson Integral

AMS Classification: 60C05, 05A30

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[†]This research has been supported by grants MTM2010-16949 and FQM5849.